PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Doug Hensley, Douglas B. West**
with the collaboration of Itshak Borosh, Paul Bracken, Ezra A. Brown, Randall Dougherty, Tamás Erdélyi, Zachary Franco, Christian Friesen, Ira M. Gessel, László Lipták, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Steven J. Miller, Richard Pfiefer, Dave Renfro, Cecil C. Rousseau, Leonard Smiley, Kenneth Stolarsky, Richard Stong, Walter Stromquist, Daniel Ullman, Charles Vanden Eynden, and Fuzhen Zhang.

Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the back of the title page. Proposed problems should never be under submission concurrently to more than one journal, nor posted to the internet before the due date for solutions. Submitted solutions should arrive before February 29, 2016. Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

11858. *Proposed by Arkady Alt, San Jose, CA.* Let D be a nonempty set and g be a function from D to D. Let n be an integer greater than 1. Consider the set X of all x in D such that $g^n(x) = x$, but $g^k(x) \neq x$ for $1 \leq k < n$. Prove that if X has exactly n elements, then there is no function f from D to D such that $f^n = g$. (Here, for $h\colon D \to D$, h^k denotes the k-fold composition of h with itself.)

11859. *Proposed by Gregory Galperin, Eastern Illinois University, Charleston, IL, and Yury Ionin, Central Michigan University, Mount Pleasant, MI.* Find all pairs (m, n) of positive integers for which there exists an $m \times n$ matrix A and an $n \times m$ matrix B, both with real entries, such that all diagonal entries of AB are positive and all off-diagonal entries are negative.

11860. *Proposed by Dimitris Vartziotis, NIKI MEPE Digital Engineering, Katsikas Ioannina, Greece.* Let ABC be a triangle. Let D, E, and F be the feet of the altitudes from A, B, and C, respectively. Extend the ray DA beyond A to a point A', and similarly extend EB to B' and FC to C', in such a way that $\sqrt{3}|AA'| = |BC|$, $\sqrt{3}|BB'| = |CA|$, and $\sqrt{3}|CC'| = |AB|$. Prove that $A'B'C'$ is an equilateral triangle.

11861. *Proposed by Phu Cuong Le Van, College of Education, Hue, Vietnam.* Let n be a natural number and let f be a continuous function from $[0, 1]$ to $\mathbb{R}$ such that $\int_0^1 f(x)^{2n+1}\,dx = 0$. Prove that

$$\frac{(2n+1)^{2n+1}}{(2n)^{2n}}\left(\int_0^1 f(x)\,dx\right)^{2n} \leq \int_0^1 (f(x))^{4n}\,dx.$$

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